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ADAPTIVE CONTROL OF A 5 DOF MAGNETIC BEARING

Introduction. Active magnetic bearings are getting more and more important in various applications. In contrast to conventional bearings the movable parts are not supported by mechanical contact or a fluid, but by magnetic forces. Therefore they are free of mechanical wear and need less maintenance. In addition, they permit a variable stiffness and damping assignment. As known, system's equilibrium is characterized by equality of magnetic and gravitational force. However, due to the attractive force between the magnet and the movable part the equilibrium is unstable and has to be stabilized by control. In industrial applications predominantly linear control laws (PID and state feedback) are used [1],[2]. As electromagnets provide a nonlinear behaviour between current and magnetic force the application of nonlinear controllers has been investigated [3],[4] with promising result.

As magnetic bearings are operating on heavy work pieces, which masses are varying in a wide range, the controller needs to be adapted. To achieve this automatically a nonlinear adaptive control scheme is presented.

Plant model of the 5 degree of freedom magnetic bearing. For the magnetic bearing 3 pairs of electromagnets (fig. 1 M1-M3) are used to support the load in 3 degrees of freedom $\left(z, \varphi_x, \varphi_y\right)$ and 3 electromagnets (fig. 1 M4-M6) for centering $\left(x, y\right)$. To measure the position in all 5 degrees of freedom 6 sensors (fig. 1 S1-S6) are used.

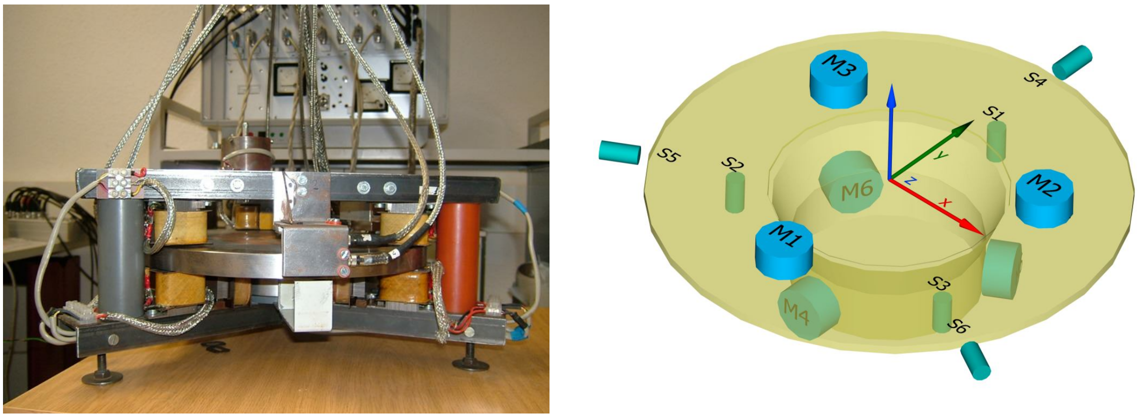

Fig.1. 5 degree of freedom magnetic bearing

The measured distances $x_S = (x_{S_1}, ..., x_{S_6})^T$ are derived from the generalized coordinates $q = (q_x, ..., q_{\varphi_y})^T$ applying a coordinate transformation $q \to x_S : x_S = J_{SB} q$. To derive the generalized coordinates from the measurement this transformation has to be inverted using the pseudo inverse $J_{SB}^+ \; : \; q = J_{SB}^+ x_S$. The generalized forces $F_q = (F_x, ..., M_y)^T$ are derived from the magnetic forces $F_A = (F_{A_1}, ..., F_{A_6})$ applying a coordinate transformation $F_A \to F_q \; : \; F_q = J_{AB}^T F_A$. To derive the actor forces from the generalized forces this transformation has to be inverted using the pseudo inverse $(J_{AB}^T)^+ : F_A = \left(J_{AB}^T\right)^+ F_q$.

The magnetic force depends on the applied current i_A and on the air gap x_A between the electromagnet and the supported disc. In this configuration a current is applied to the upper electromagnet in order to generate a positive force and to the lower to generate a negative force respectively. Here a positive force $F \geq 0$ is associated with a positive current $i \geq 0$ and a negative force $F < 0$ with a negative current $i < 0$ respectively.

$$F_k(i_A, x_A) = \begin{cases} a \dfrac{i^2}{\left(k_{1o} - 2x_A\right)^2} & \text{for} \quad i \geq 0 \\ -a \dfrac{i^2}{\left(k_{1u} - 2x_A\right)^2} & \text{for} \quad i < 0 \end{cases} \qquad \text{with } k = 1, ..., 3 \tag{1}$$

Assuming that the associated mass matrix M has diagonal structure the overall systems equations are as follows:

$$\begin{pmatrix} \dot{x}_q \\ \ddot{x}_q \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix}}_{A} \begin{pmatrix} x_q \\ \dot{x}_q \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ I \end{pmatrix}}_{B} \left(-g_E + M^{-1} J_{AB}^T F_A \right) \text{ with } M = \begin{pmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & J_{xx} & 0 \\ 0 & 0 & 0 & 0 & J_{yy} \end{pmatrix} \text{ and } g_E = \begin{pmatrix} 0 \\ 0 \\ -g \\ 0 \\ 0 \end{pmatrix}. \tag{2}$$

Nonlinear Adaptive Control. In the following feedback linearization control theory is applied assuming the load would be known. Therefore the system has to be transformed to the Brunovsky form, where all nonlinearities can be exactly compensated. The remaining chain of integrators can then be stabilized applying linear control theory. The unknown load will then be substituted by its estimate. The derived control is therefore a certainty equivalence law [5],[6] which has to be augmented by an appropriate parameter update law to account for the unknown and changing load. The parameter update law will be derived applying Lyapunov stability theory, which guarantees the stability of the closed loop system for a wide range of masses.

For the 5 degree of freedom magnetic bearing the control task is set point regulation. For the sake of clarity the generalized forces F_q will be treated as the control input u. The associated u is derived from (1). Fortunately the systems equations are already in the Brunovsky form (2). Using the associated error variable $z_q = \left(x_q - x_{qS} \quad \dot{x}_q \right)^T$ linear optimal control theory can be used to derive a stabilizing control law. Therefore the following Riccati equation (3) is solved.

$$A^T P + PA - PBR^{-1}BP + Q = 0 \tag{3}$$

$$v = -R^{-1}B^T Pz \tag{4}$$

The solution P can be used to construct a Lyapunov function $V = \frac{1}{2} z^T Pz$. Compensating the gravity acceleration and the unknown mass matrix gives the following certainty equivalence control law u:

$$u = \hat{M}\left[g_E + v \right]. \tag{5}$$

To derive a parameter update law which guarantees stability in the sense of Lyapunov the Lyapunov function $V = \frac{1}{2} z^T Pz$ has to be augmented by an additional term $\frac{1}{2\gamma} \sum_{i=1}^{5} \tilde{\rho}_i^2$,where $\tilde{\rho}_i = \rho_i - \hat{\rho}_i$ is the estimation error and $\rho_{1,2,3} = \frac{1}{m}$, $\rho_4 = \frac{1}{J_{xx}}$ and $\rho_5 = \frac{1}{J_{yy}}$ are parameters of the mass matrix.

$$V = \frac{1}{2} z^T Pz + \frac{1}{2\gamma} \sum_{i=1}^{5} \tilde{\rho}_i^2 \tag{6}$$

Time differentiation gives:

$$\dot{V} = z^T P\dot{z} + \frac{1}{\gamma} \sum_{i=1}^{5} \tilde{\rho}_i \dot{\tilde{\rho}}_i \text{ ,} \tag{7}$$

$$\dot{V} = z^T PAz + z^T PB\left[-g_E + M^{-1}u \right] - \frac{1}{\gamma} \sum_{i=1}^{5} \tilde{\rho}_i \dot{\hat{\rho}}_i \text{ .} \tag{8}$$

Completing by zero ($0 = g_E + v - \hat{M}^{-1} u$ (5)) yields:

$$\dot{V} = z^T PAz + z^T PB\left[-g_E + M^{-1}u + g_E + v - \hat{M}^{-1} u \right] - \frac{1}{\gamma} \sum_{i=1}^{5} \tilde{\rho}_i \dot{\hat{\rho}}_i \text{ ,} \tag{9}$$

$$\dot{V} = z^T PAz + z^T PBv + z^T PB\left[M^{-1}u - \hat{M}^{-1} u \right] - \frac{1}{\gamma} \sum_{i=1}^{5} \tilde{\rho}_i \dot{\hat{\rho}}_i \text{ ,} \tag{10}$$

$$\dot{V} = z^T PAz - z^T PBR^{-1}B^T Pz + \sum_{i=1}^{5} \tilde{\rho}_i \left[u_i B^T Pz - \frac{1}{\gamma}\dot{\hat{\rho}}_i \right]. \tag{11}$$

To compensated the unknown dynamics caused by the estimation error $\tilde{\rho}_i$ the parameter update law is chosen as follows:

$$\dot{\hat{\rho}}_i = \gamma u_i B^T Pz. \tag{12}$$

Applying the parameter update law (12) guarantees the negative definiteness of the time derivative of the Lyapunov function and therefore stability in the sense of Lyapunov:

$$\dot{V} = -\frac{1}{2} z^T Qz - \frac{1}{2} z^T PBR^{-1}B^T Pz \le 0. \tag{13}$$

To verify the presented control scheme a mass m=2kg was added at set point. The control parameters Q and R are chosen as follows:

$$\begin{aligned} &q_{11} = q_{33} = q_{55} = q_{77} = q_{99} = 10^7, \\ &q_{22} = q_{44} = q_{66} = q_{88} = q_{1010} = 10^3, \\ &r_{11} = r_{22} = r_{33} = r_{44} = r_{55} = 1. \\ &q_{ij} = r_{ij} = 0 \text{ for } i \ne j. \end{aligned} \tag{14}$$

In the upper part of (fig. 2) the position q_z (fig. 2 left) and the applied current i_{A_1} (fig. 2 right) are shown for the feedback linearization controller without adaptation $\gamma = 0$. In the lower part of (fig. 2) the position q_z (fig. 2 left) and the applied current i_{A_1} (fig. 2 right) are shown for the proposed adaptive control law with $\gamma = 0.1$.

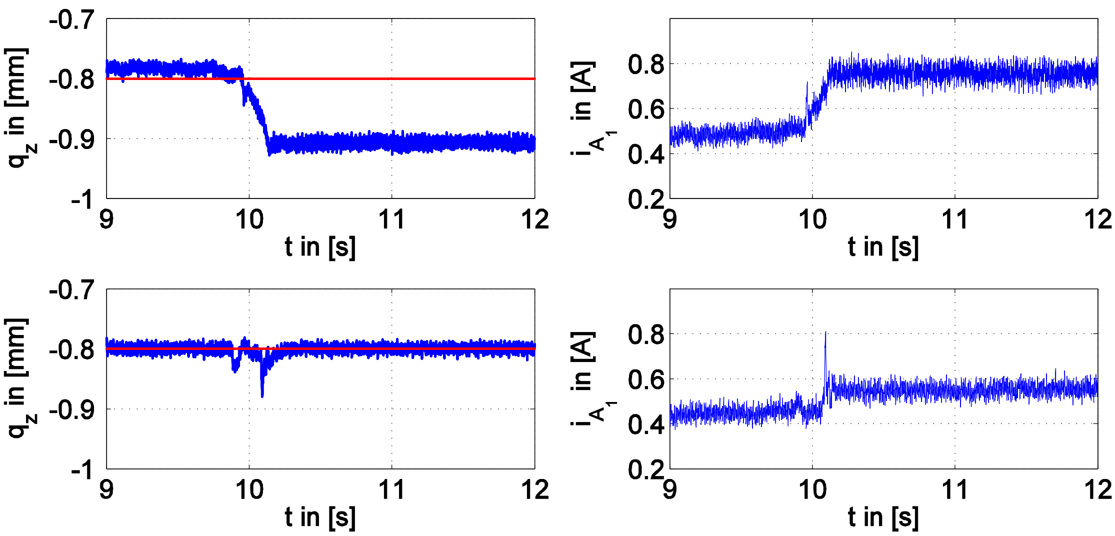

Fig.2. Position q_z (left) and current i_{A_1} (right) for feedback linearization (top) and adaptive feedback linearization (bottom) (m=2kg)

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