Optimisation of economic system

OLIYNYK V.

SUMY STATE UNIVERSITY

The three-period model of economic system with one generalised product and with final quantity of economic subjects is considered in the work . The using of units of production by each agent can happen during the different moments of time and has casual character. Some chances of reception of profit on fulfillment of financial operations are considered. The optimum effect from product consumption is reached at maximisation of function of utility taking into account restrictions. Research of various variants of financial interaction between agents is made. Some numerical results are received.

Keywords: the financial agent; the economic model; the utility function; the probability; the discounting factor

В работе рассмотрена трехпериодная модель экономической системы с одним обобщенным продуктом и с конечным количеством экономических субъектов. Использование единиц продукции каждым агентом может происходить в разные моменты времени и носит случайный характер. Рассматривается несколько возможных случаев получения прибыли от совершения финансовых операций. Оптимальный эффект от потребления продукта достигается при максимизации функции полезности с учетом ограничений. Проводится исследование различных вариантов финансового взаимодействия между агентами. Получены некоторые численные результаты.

В роботі розглядається трьохперіодна модель економічної системи з одним загальним продуктом та кінцевою кількістю економічних суб'єктів. Використання одиниць продукції кожним агентом може відбуватися в різні моменти часу та носить випадковий характер. Розглядається декілька можливих випадків отримання прибутку від здійснення фінансових операцій. Оптимальний ефект від використання продукту досягається при максимізації функції корисності з урахування обмежень. Здійснюється дослідження різних варіантів фінансових взаємодій між агентами. Отримані деякі чисельні результати.

Introduction. Let we have economic system in which there is a businessman who can give the project as object for investment. Final well-being of the businessman is estimated by means of utility function. The problem is to increase this function at the expense of available resources.

As the subject who will execute this function, the financial intermediary, namely bank can act. In regular intervals distributing risks, with increase of an amount of operations, there is a possibility to receive profit at the expense of different types of economy. Thus, banks help private persons to diversify the savings with possibility to receive more benefit, than simply to receive percent from the contribution. Financial intermediaries actually create new financial actives.

In [1] the financial object which specialises on acquisition and sale of financial contracts and securities is meant financial intermediaries. According to this definition as financial intermediaries brokers and the dealers working in the financial market, and also banking establishments can act. Banks differ from other financial intermediaries with the specific features:

ι) have matters with such forms of financial contracts (have placing of credits and deposits), which less liquid, than the active securities;

ιι) the characteristic of the depositary contracts concluded by banks with lenders, qualitatively differs from characteristics of the credit contracts concluded by them with borrowers.

These singularities give the chance to consider banks as the economic institutes, financial contracts carrying out transformation. Such approach has developed in works of known economists such as Benston G., Smith C.W. [2] and Fama E. [3]. Banks accumulate in themselves resources and can use means for investment profitable, but with low level of current liquidity, projects. They can represent itself as financial intermediaries until investors simultaneously want to take advantage of the means.

Problem statement. Let's examine abstract three-period model of economy $\left(t_0, t_1, t_2\right)$, with one generalised product [4]. Within the limits of this model the final set of agents (economic subjects) functions. We will examine two cases.

$$P^{**} = I_0 L_0 + (I_1 K_2 / K_1 + I_2) L_2 = $$
$$= I_0 L_0 + I_2^* L_2$$

A. Let in point of time t_0 each of agents possesses one unit of a product which brings in the fixed income. Each of agents can take a share from the generalised product in number of $I = I_1 + I_2$ and to invest share I_1 - in the point of time t_1 (before project end), and share I_2 - in point of time t_2 (after project end). Profit P_i on investment during the different moments of time t_i finds by the formula:

$$P_i = \mu_i C_i L_i = I_i L_i \qquad (i = 0,1,2) \tag{1}$$

Where μ_i - probability of use of a unit of production in the point of time t_i; C_i - amount of units of production; I_i - shares from the generalised product; L_i - profit of a unit of production.

B. We will suppose, that between agents there is a trade and they can use it for magnification of the well-being. The assumption about a trade possibility in the given model means, that in point of time t_1 the financial market on which agents can exchange the product on risk-free the bond opens. Further, in point of time t_2, under the bond it is possible to receive a yield quantity.

© Oliynyk V., 2011

Let's designate through K_i a bond course in points of time t_i accordingly. According to it, profit of the agent with early consumption term can be calculated with formula equation:

$$P^* = I_0 L_0 + I_1 L_1 / K_1 + I_2 L_2 \quad (2)$$

In (2) it is supposed, that the agent in point of time t_1 will exchange a share of yield I_1 for the bond, share I_2 invests in the project in point of time t_2.

If the agent is characterised by late term of consumption he in point of time t_1 on share I_1 purchases bonds at the rate of K_1, and in the point of t_2 - sells them at the rate of K_2. The profit on this operation makes:

$$P^{**} = I_0 L_0 + (I_1 K_2 / K_1 + I_2) L_2 = I_0 L_0 + I_2^* L_2 \quad (3)$$

The effect from yield consumption generally can be evaluated by means of some utility function $U(C)$.

The expectation of the general usefulness is in a kind:

$$U(C_0; C_1; C_2) = \sum_{i=0}^{2} \mu_i \rho_i U(C_i) \quad (4)$$

Where ρ_i - a discount factor.

Let's examine case **A**, when there are no connections between agents. Each agent, independently one from other, can invest a share from the amount of a product in the given project. We will suppose, that the project is infinitely a dividend and is not set initial boundary on magnitude of the means invested in it.

Thus, for deriving of optimum distribution of use of a product, it is necessary to solve the following problem of optimisation:

To maximise function of utility (4) with restrictions

$$\begin{cases} \sum_{i=0}^{2} \mu_i = 1 \\ \sum_{i=0}^{2} \mu_i C_i = \sum_{i=0}^{2} I_i = 1 \\ C_i; \rho_i; \mu_i \geq 0 \end{cases} \quad (5)$$

At the account of commercial relations (problem **B**), from the point of view of maximisation by the agent of function of utility, optimum distribution of a product can be received as a finding of function of the purpose (4) at following restrictions:

1. For early consumption

$$\begin{cases} \sum_{i=0}^{2} \mu_i = 1 \\ \mu_0 C_0 + \mu_1 C_1 / K_1 + \mu_2 C_2 = 1 \\ C_i; \rho_i; \mu_i \geq 0 \end{cases} \quad (6)$$

2. For late consumption

$$\begin{cases} \sum_{i=0}^{2} \mu_i = 1 \\ \mu_0 C_0 + \mu_1 C_1 K_2 / K_1 + \mu_2 C_2 = 1 \\ C_i; \rho_i; \mu_i \geq 0 \end{cases} \quad (7)$$

The problem solution. Generally, when it is impossible to express one variable through another, type (4) problem - (5) it is possible to solve by means of function of Lagrange:

$$F = U(C_0; C_1; C_2) - \lambda\left(\sum_{i=0}^{2} \mu_i C_i - 1\right) \quad (8)$$

Where λ - multiplier of Lagrange.

The condition of an extremum of function (8) is reduced to the system solution:

$$\begin{cases} \rho_1 U'(C_1) = \rho_2 U'(C_2) \\ \sum_{i=0}^{2} \mu_i C_i = 1 \end{cases} \quad (9)$$

For an example, utility function can be taken in the form of function of type of Neumann-Morgenstern:

$$U(C) = 1 - \exp(-aC) \quad (a = const,\ a > 0) \quad (10)$$

With allowance for (10) decision of a problem (9) looks like:

$$\begin{cases} C_1 = C_2 - \ln(\rho_2 / \rho_1) / a \\ C_2 = [(1 - I_0) + \mu_1 \ln(\rho_2 / \rho_1) / a] / (1 - \mu_0) \end{cases} \quad (11)$$

The condition of positivity of an amount of units of production is displayed in the form of restrictions on discounting factors:

$$\exp(-a(1 - I_0) / \mu_1) \leq \rho_2 / \rho_1 \leq \exp(a(1 - I_0) / \mu_2) \quad (12)$$

At a finding of optimum distribution of a product, with allowance for commercial relations between agents, we solve problems (4), (6) and (4), (7) for early and late type of consumption accordingly.

The decision of a problem of late consumption looks like (if $K_1 = 1$ we have a problem of early term of consumption):

$$\begin{cases} C_1 = C_2 - \ln[\rho_2 K_2 / (\rho_1 K_1)] / a \\ C_2 = \{(1 - I_0) + \mu_1 K_2 \ln[\rho_2 K_2 / (\rho_1 K_1)] / (a K_1\} / \\ \quad / (\mu_1 K_2 / K_1 + \mu_2) \end{cases} \quad (13)$$

with restrictions:

$$\begin{aligned} &\exp(-a(1 - I_0) / \mu_1) \leq \rho_2 K_2 / (\rho_1 K_1) \leq \\ &\leq \exp(a(1 - I_0) / \mu_2) \end{aligned} \quad (14)$$

Numerical examples. Let's suppose, that commercial relations between agents are absent.

1. Let $I_0 = 0{,}1;\ \mu_0 = 0{,}2;\ \rho_1 = 0{,}9;\ \rho_2 = 0{,}4$. The figure 1 characterises the most rational distribution of a generalised function of utility depending on probability of investment in different points of time and for different significances of parametre a.

2. We will suppose, that $I_0 = 0;\ \mu_0 = 0;\ \rho_1 = 0{,}8;\ \rho_2 = 0{,}5;\ K_1 = 0{,}9;\ a = 1$. On figure 2 optimum distribution of an amount of units of production C_i with early and late term of consumption ($K_2 = 2$) is shown. To parametre C_1 there correspond curves 1 and 4, early and late terms of consumption accordingly, and to parametre C_2 - curves 3 (early term) and 2 (late term). On figure 3 distribution of a utility function (4) is shown at various significances of parametre K_2.

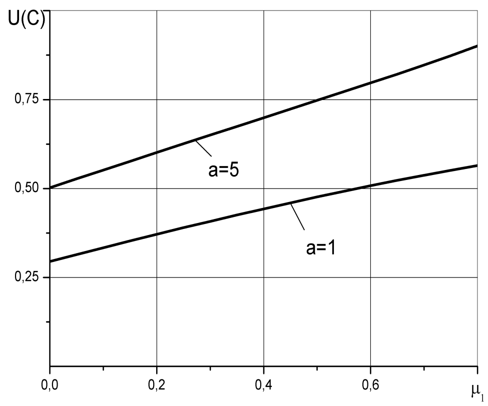

Figure 1: Distribution $U(C_0;C_1;C_2)$.

Let's consider a case when there are commercial relations between agents.

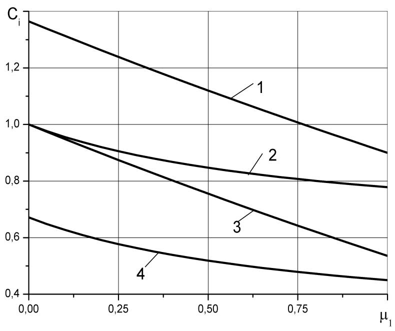

Figure 2: Distribution C_i

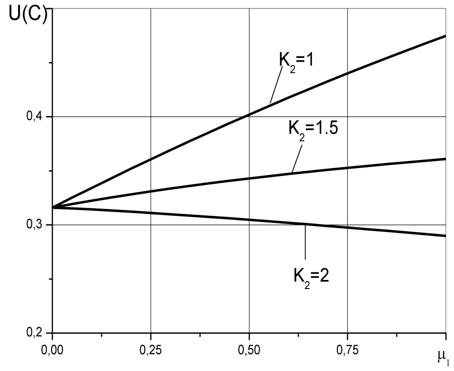

Figure 3: Distribution $U(C_1, C_2)$.

Conclusion

On the basis of the received outcomes it is possible to make a platoon, that the kind of optimum distribution of an amount of units of production in different points of time, does not depend on an amount of units of production which remains in an initial point of time. Depending on consumption type, there is a qualitative redistribution of an optimum amount of the units of production, leading to utility function maximisation.

REFERENCES

1. Freixas X., Rochet J.-Ch. (1998). Microeconomics of banking. MIT Press,
2. Benstone G., Smith C.W.(1976). A transaction cost approach to the theory of financial intermediation. *The Journal of Finance*, vol.31, pp.215-231.
3. Fama E. (1980). Banking in the theory of finance. *Journal of Monetary Economics,* vol. (6)1, pp.39-57.
4. Bryant J. (1980). A model of reserves, bank runs and deposit insurance. *Journal of Banking and Finance,* vol.4.issue 4, pp.335-344.

пост. 12.05.11